

Is 'genomgång och bok' the only way
to teach mathematics?

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Genomgång och bok

- ❑ In a recent interview study we asked Swedish upper secondary students to describe their typical mathematics lesson.
- ❑ Their response was typically *genomgång och bok*
 - ❑ Normally we go through what we will be doing; first we have a short summary of what we will do, then the teacher usually holds a genomgång on the section we are going to work with... And then we usually undertake exercises on what we have learned

- ❑ ...first the teacher talks about the chapter we are moving into. And then we work by ourselves mostly from the books, doing different exercises and so on... That's pretty much it. And so we work through... we follow the book, like chapter after chapter. Yeah, the teacher often has a genomgång... on what we're doing.
- ❑ “the teacher gives a kinda introduction to what we're going to do, and really then we just do exercises (...) That is it... and when we are done with the exercises we can just go out”.
- ❑ I listened to him quite a lot and I answered a lot of the questions he asked but (...) I actually didn't try that much because I slept a lot in the lessons (...) and then we might just start working from the book... You could go outside and sit if you wanted a more quiet place. So that was very nice. But we worked a lot from the books.

On genomgång in particular

- ❑ He usually opens the book and opens the page that we're going to read about or the chapter, and he starts to talk about the different kinds of things that have to do with this. And he writes it all down on the white board.
- ❑ He starts showing us this new method that is part of the new chapter that we're moving into... We usually listen and are free to take notes, but you don't have to. So, you listen as good as you can, I guess, and follow and try to understand...
- ❑ I listen while I take notes. I write exactly what he is writing, so it memorises my brain better...
- ❑ Yeah, I usually just listen, because I don't like writing so much... But usually I just listen and then take notes.
- ❑ He writes an example on the board, an equation for example, and then he goes through the different rules that apply to solving the equation... And then he may give us an example to do together and if there is someone who wants to go forward, for example, to the board, you can go and report how you do it

How long does genomgång last?

- ❑ For Martin a genomgång is twenty minutes of a one-hour lesson
- ❑ Hans believed it typically lasts “fifty to fifty-five minutes and then we get five minutes for exercises”.
- ❑ For Jamie it was usually “about a half-hour”,
- ❑ For Monika, “I think he talks for between twenty and thirty minutes”,
- ❑ While for Naomi,

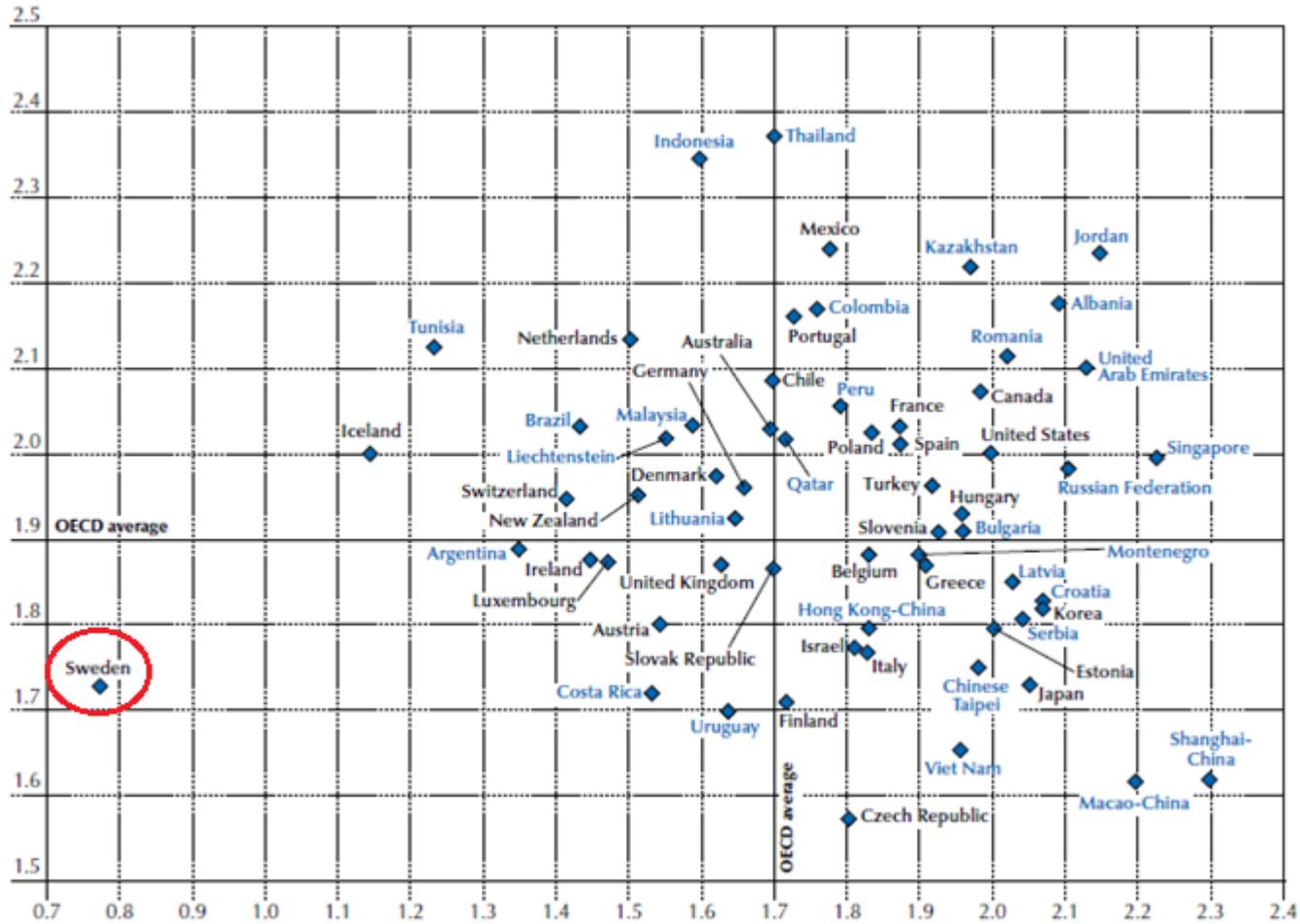
It depends on how much time we need to think about things, because sometimes it takes up most part of the lesson like 30 or 35 minutes. But that's just because he really wants us to think about it... But when it's smaller tasks ... it's like 15 minutes.

An interesting (and important) aside

- ❑ PISA 2012 says that Swedish students are very poor mathematically
- ❑ PISA 2012 says that Swedish students know very little about linear equations
- ❑ Press reactions to PISA 2012 were predictable
 - ❑ Dagens Nyheter: *Sweden is the worst in the class* (Sverige sämst i klassen)
 - ❑ Aftonbladet: *the Swedish school is sinking* (Svensk skola sjunker)
- ❑ DN also reported comments from leading academics
 - ❑ Jakobsson: We should investigate the policy of parental choice
 - ❑ Pettersson: We need a crisis commission
- ❑ But are things really that bad?
- ❑ No, they are not... But they could be better

PISA and Swedish opportunities to learn

Exposure to applied mathematics



Exposure to formal mathematics

How can this be interpreted?

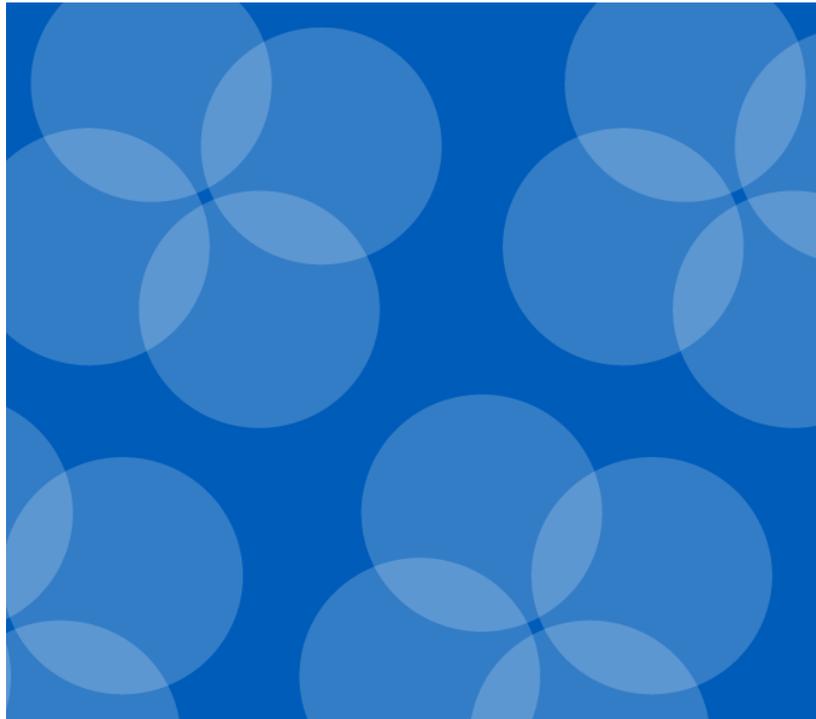
- ❑ Do Swedish teachers deliberately teach neither formal mathematics or applications?
- ❑ Does it mean that both algebra and mathematical applications are missing from the Swedish curriculum?
- ❑ Does it mean that what Swedish teachers teach is determined by national tests that do not address such matters?
- ❑ Does it mean that Swedish teachers stick too closely to textbooks that do not address such matters?
- ❑ Or are there other things going on that might help us to understand better?

Does PISA measure what students know?

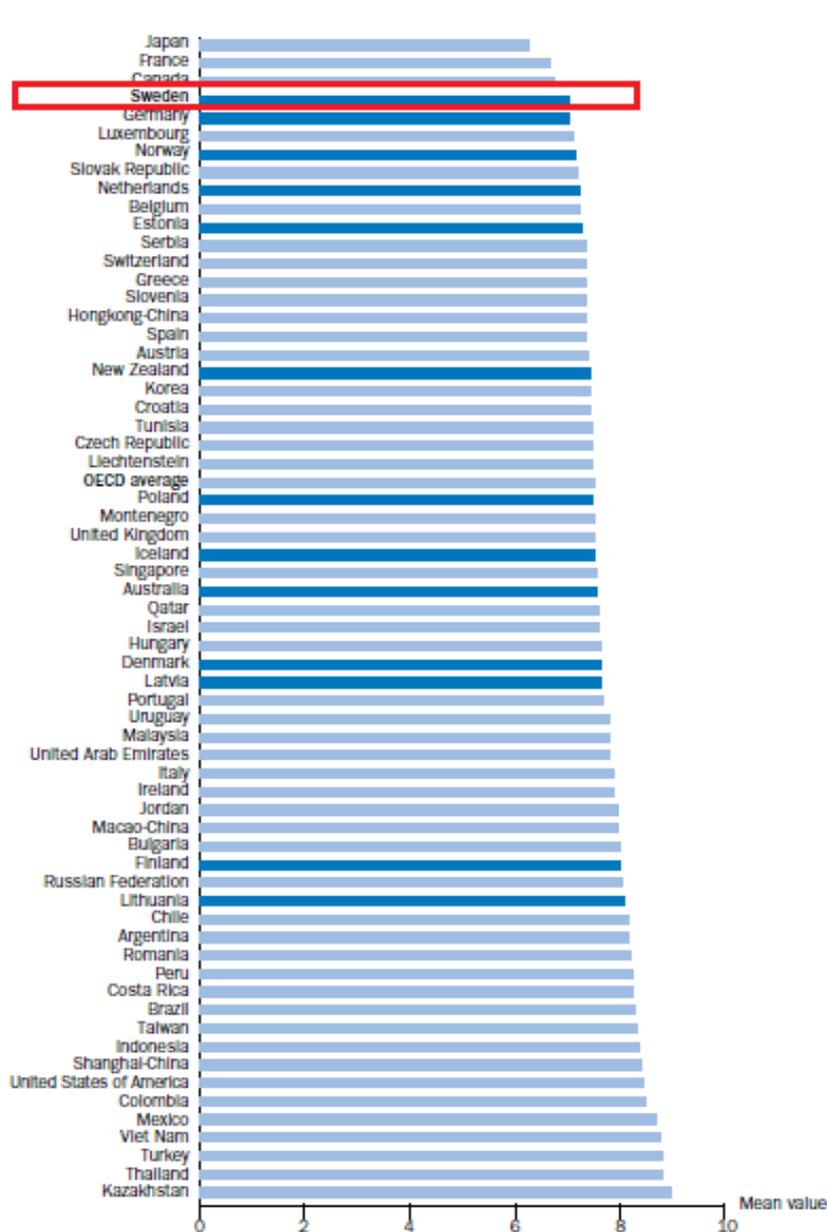
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To respond or not to respond

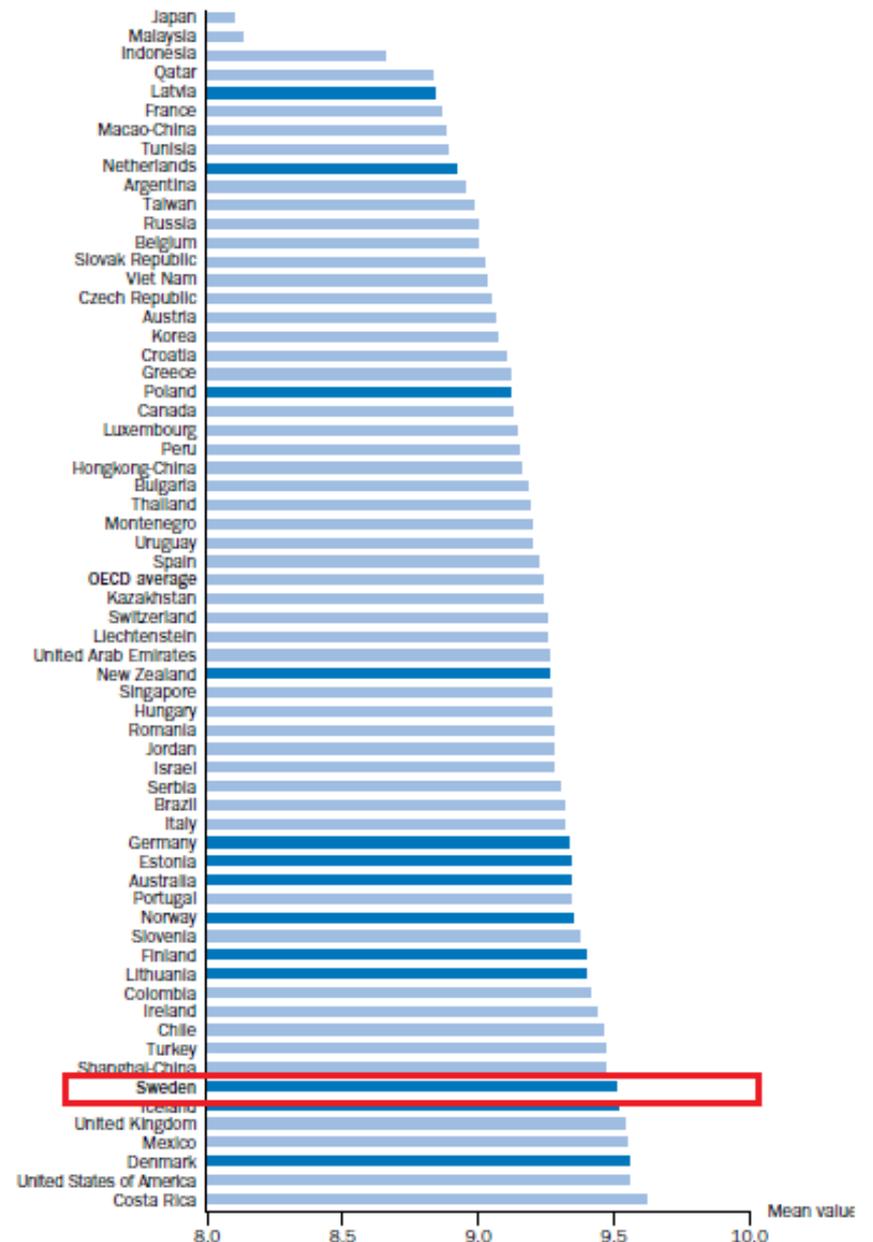
The motivation of Swedish students in taking the PISA test

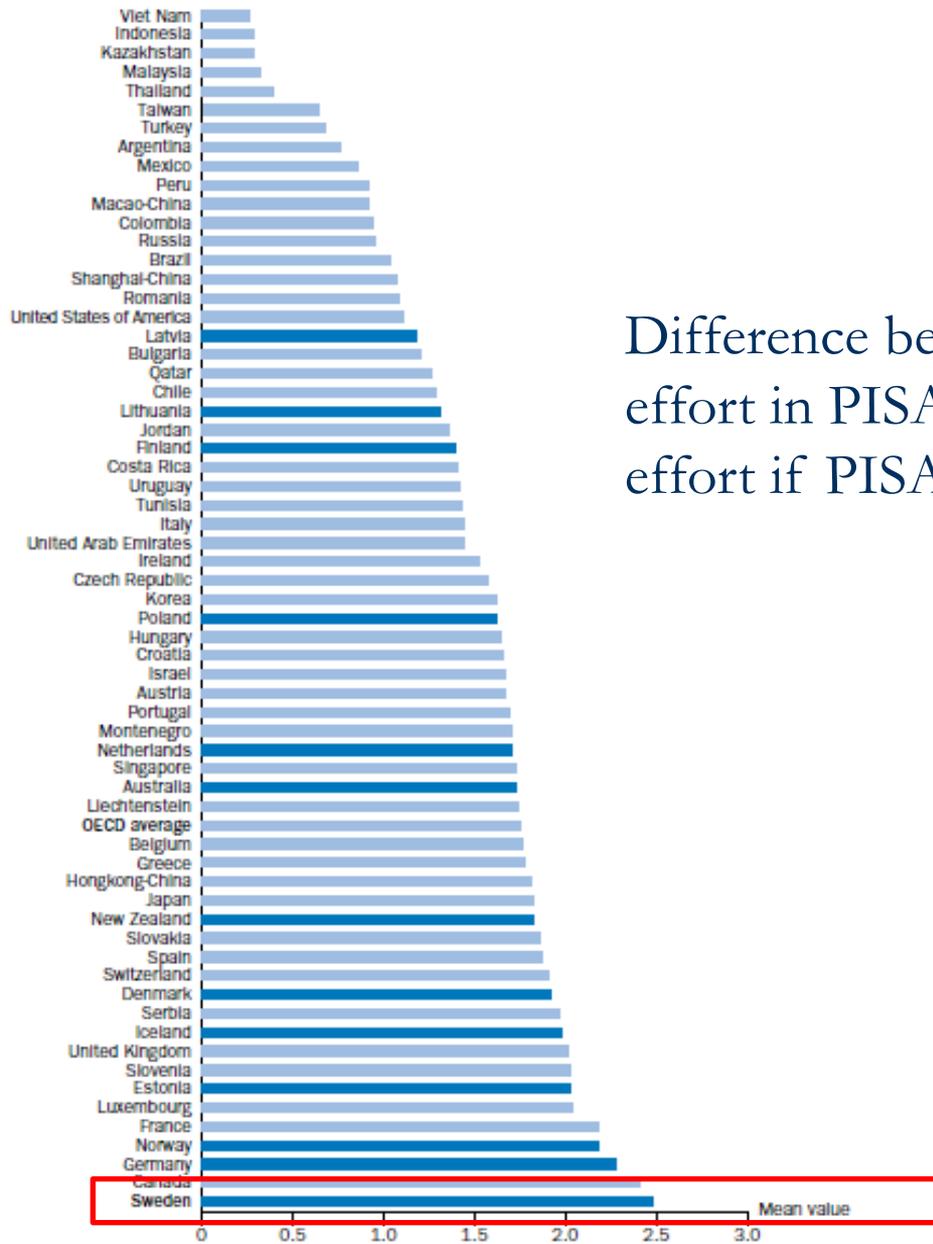


Mean value of reported effort in PISA 2012.



Mean value of estimated effort if PISA was going to count towards a grade





Difference between average reported effort in PISA and average estimated effort if PISA were to count for anything.

Effort over time

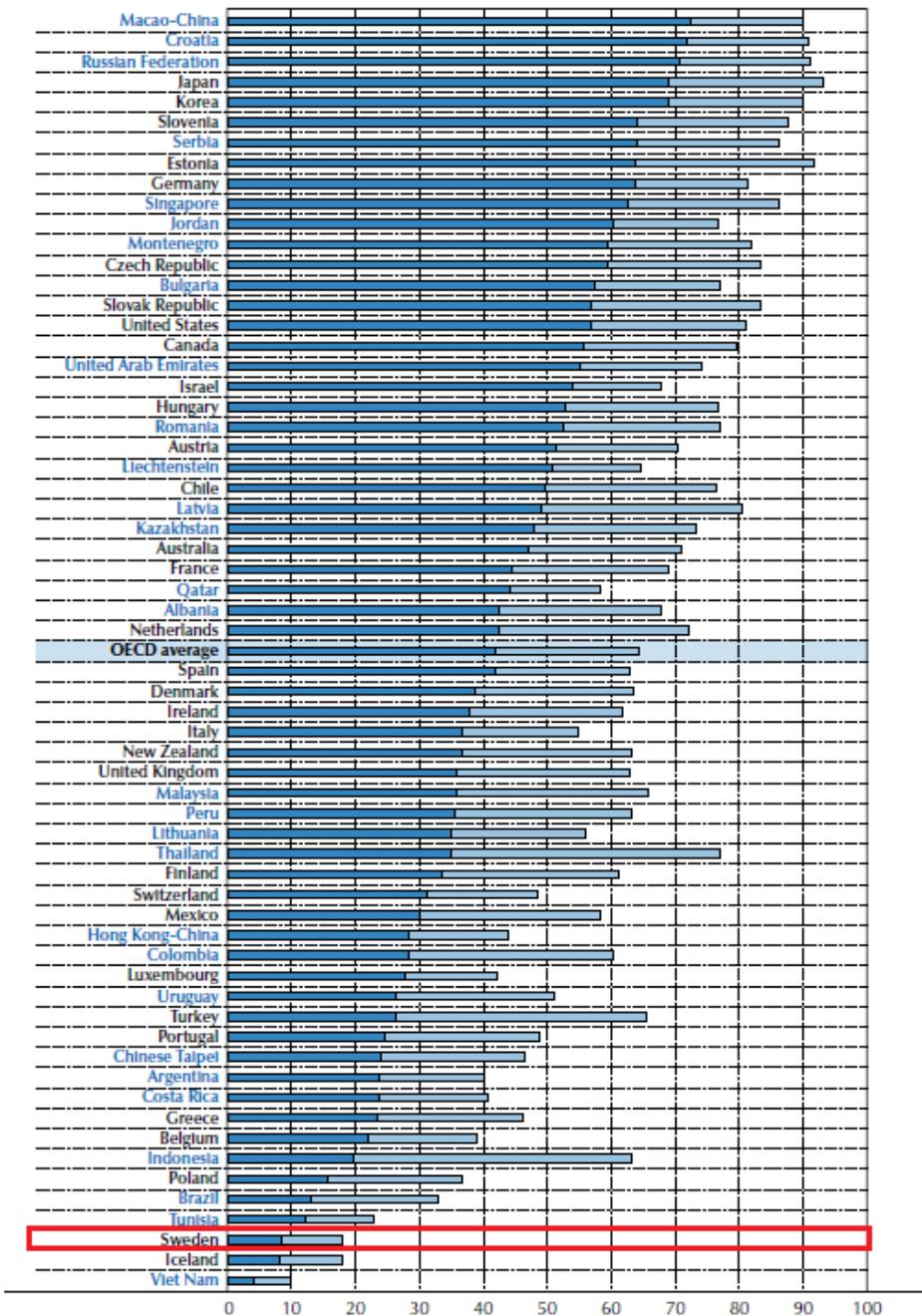
PISA	2000	2003	2006	2009	2012
Mathematics	510	509	502	494	478
Effort		7.38			7.03

Now, the drop on the effort score from 7,38 to 7,03 is 0,35.

This is a drop of $\frac{100 \cdot 0,35}{7,38} = 4,74\%$

What would a drop of 4,74% on the PISA 2003 mathematics score look like? That is, can we predict the PISA score for 2012?

Well, to do this we calculate 95,26% of the 2003 score, which gives 485.



What about linear equations?

$$3x+5=17$$

- They understand the concept
- They have heard of it often



Can we trust PISA: Linear equations

- We asked Swedish upper secondary students, “how would you explain this to someone who missed your lessons on equations?”

$$x + 5 = 4x - 1$$

$$5 = 3x - 1$$

$$6 = 3x$$

$$2 = x$$

Results

- ❑ Students explained the solution clearly and with confidence.
- ❑ They explained the nature of x as an unknown that needed to be found
- ❑ They understood that non-arithmetical equations cannot be solved by a process of operation reversal
- ❑ They had a relational understanding of the equals sign
- ❑ They spoke of ‘do the same to both sides’
- ❑ They rejected ‘change the side change the sign’
- ❑ In a second study, their accounts were much more sophisticated than Cypriot beginning primary teachers

Alternatives to genomgång och bok?

- ❑ To explore this I shall look at two lessons, one from Flanders and one from Hungary. Both focus on introducing linear equations.
- ❑ We see Pauline in Flanders and Emese in Hungary
- ❑ I am not presenting them as faultless; we can always criticise



Pauline's ongoing teaching

- ❑ Pauline began her third lesson by going through homework set the previous lesson.
- ❑ One of the problems was $4(x + 15) = 180$.
- ❑ The solution was discussed publicly with Pauline reminding her students of her expectations with respect to annotations. The final solution appeared as below

$$4(x + 15) = 180$$

Eliminate brackets

$$4x + 60 = 180$$

Get x terms to one side and numbers to the other

$$4x = 180 - 60$$

Calculate where possible

$$4x = 120$$

Divide both sides by 4

$$x = 30$$

- ❑ However, the above reflected a well defined and inflexible sequence of activity
- ❑ The complexity of the equations increased during the final two lessons, with the test, given at the end of the fifth lesson, comprising three problems.

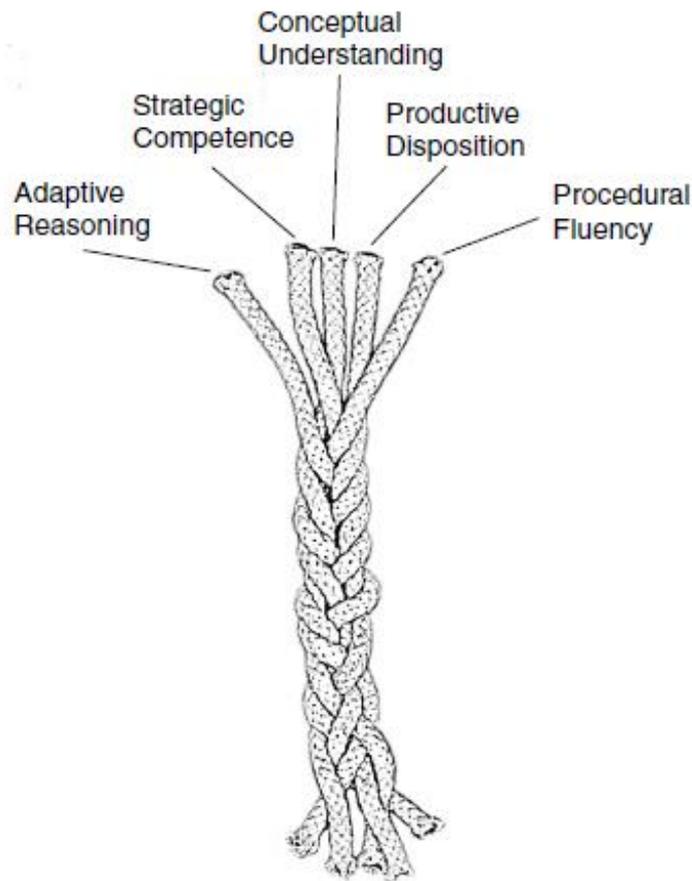
- ❑ $5(p + 2) = 6p - 3$; $(14 - 2x) - (x + 12) = x - 2$; $-3/4y = -2/3$.

Emese' ongoing teaching

- ❑ Emese spent the last three lessons of her sequence working on equations drawn from the world of mathematics and *realistic* word problems.
- ❑ A typical example of a mathematical world equations was
$$15 - \{1 - 2[x - (3 - x)]\} = 72$$
- ❑ This was particularly interesting in the way Emese encouraged two approaches to the solution
- ❑ A typical word problem was *A stake is driven through a pond into the ground. If 1/4 of the stake's length is in the ground, 2/5 is in water and 2.8m is above the water, how long is the stake?*
- ❑ In every case, a problem was posed and a short period of individual working was followed by a public discussion of the solution.
- ❑ In such problems, which were clearly non-trivial, Emese ensured that students worked constantly with brackets, negatives and fractions.
- ❑ Finally, how do such matters fit against international expectations of mathematics teaching?



The five strands of mathematical proficiency



Conceptual understanding: An integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods.

Procedural fluency: Knowledge of procedures, and when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Strategic competence: The ability to formulate mathematical problems, represent them, and solve them

Adaptive reasoning: The capacity to think logically about the relationships among concepts and situations, and how to justify the conclusions.

Productive disposition: The belief that mathematics as both useful and worthwhile; to believe that effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics

In the world's best mathematics classrooms

- ❑ Mathematics is acknowledged as difficult but worthy of study
- ❑ Mathematics is viewed as a problem-solving activity
- ❑ Problems are chosen to exemplify mathematical generality
- ❑ Teachers do not seek to reduce the complexity of mathematical ideas
- ❑ Teachers do not shy from the vocabulary of mathematics
- ❑ Mathematics lessons are sequenced to emphasis coherence and continuity
- ❑ Students are expected to engage with proof and justification
- ❑ The applications of mathematics are subordinated to the subject itself
- ❑ Mathematical ideas are revisited constantly within the problems offered
- ❑ Relatively little time is given to the practice of routine procedures or exercises
- ❑ I am not convinced that conventional genomgång och bok achieve these, but am now working with colleagues in Stockholm on addressing this.